Exercise 69

Suppose that

$$f(1) = 2 \qquad f'(1) = 3 \qquad f(2) = 1 \qquad f'(2) = 2$$
$$g(1) = 3 \qquad g'(1) = 1 \qquad g(2) = 1 \qquad g'(2) = 4$$

(a) If $S(x) = f(x) + g(x)$, find $S'(1)$.

(b) If $P(x) = f(x)g(x)$, find $P'(2)$.

(c) If $Q(x) = f(x)/g(x)$, find $Q'(1)$.

(d) If $C(x) = f(g(x))$, find $C'(2)$.

Solution

Part (a)

Find a formula for $S'(x)$.

$$\begin{aligned} S'(x) &= \frac{d}{dx}[f(x) + g(x)] \\ &= \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)] \\ &= f'(x) + g'(x) \end{aligned}$$

Plug in $x = 1$ to find $S'(1)$.

$$S'(1) = f'(1) + g'(1) = 3 + 1 = 4$$

Part (b)

Find a formula for $P'(x)$.

$$\begin{aligned} P'(x) &= \frac{d}{dx}[f(x)g(x)] \\ &= \left[\frac{d}{dx}f(x)\right] g(x) + f(x)\left[\frac{d}{dx}g(x)\right] \\ &= f'(x)g(x) + f(x)g'(x) \end{aligned}$$

Plug in $x = 2$ to find $P'(2)$.

$$P'(2) = f'(2)g(2) + f(2)g'(2) = (2)(1) + (1)(4) = 6$$

Part (c)

Find a formula for $Q'(x)$.

$$\begin{aligned} Q'(x) &= \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] \\ &= \frac{\left[\frac{d}{dx}f(x)\right]g(x) - \left[\frac{d}{dx}g(x)\right]f(x)}{[g(x)]^2} \\ &= \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2} \end{aligned}$$

Plug in $x = 1$ to find $Q'(1)$.

$$Q'(1) = \frac{f'(1)g(1) - g'(1)f(1)}{[g(1)]^2} = \frac{(3)(3) - (1)(2)}{(3)^2} = \frac{7}{9}$$

Part (d)

Find a formula for $C'(x)$.

$$\begin{aligned} C'(x) &= \frac{d}{dx}\left[f(g(x))\right] \\ &= f'(g(x)) \cdot \left[\frac{d}{dx}g(x)\right] \\ &= f'(g(x)) \cdot g'(x) \end{aligned}$$

Plug in $x = 2$ to find $C'(2)$.

$$C'(2) = f'(g(2)) \cdot g'(2) = f'(1) \cdot 4 = 3 \cdot 4 = 12$$